ISSN 2229-5518

Synchronizing Chaotic Systems CHEN-LUby Fractional Order Using Active Sliding Mode Controller and Optimizing Parameters by PSO Algorithm

Toktam motamedifar, Mahdi yaghoobi

Abstract— Chaotic behavior of dynamic systems can be observed in many real applications such as circuits - power systems – medicine – electrochemical biology. Chaos is one of the most interesting subjects attracted specialists' attention in different scientific areas. This research is aimed at studying chaos in fractional order systems CHEN – Lu. A sliding mode controller is then considered for synchronizing fractional order chaotic systems with based and follower structure. For optimization ofthe parameters of this controller, particle swarm optimization (PSO) algorithm is used. Finally, the numerical simulation shows that this method needs less time for synchronization than similar methods.

Index Terms— Active Sliding Mode Controller, Chaotic Systems CHEN-LU, Fractional Order, Synchronization, PSO

1.Introduction

Fractional differential calculus relates back to about 300 years ago. However, its applications in physics and engineering have started in recent decades. Many systems in interdisciplinary areas can be modeled by fractional order derivatives [1]. Controlling and synchronizing chaotic systems have been one of the most interesting subjects in recent years and attracted scientists' attention.
For example, in [2], synchronization of the integrated fractional order chaotic systems has been studied. [4] presents control based on active sliding mode controllerfor synchronization of fractional order chaotic system. [5] uses fractional Routh-Horowitz conditions for controlling fractional order chaos in Duffing-VandePol system. In [6], a smart fractional sliding surface has been defined and a sliding controller has been studied for a nonlinear system. The new fractional order hyper-chaotic system has been presented in [7] and designed by placing pole for synchronizing a class of non-linear fractional order systems. [8] studies the coordination between fractional order chaotic system. A simple but efficient method has been presented in [9] for controlling the fractional chaotic system using T-S fuzzy model and an adaptive regulation mechanism.

2. Systems with Fractional Derivatives

- *Department of Electrical Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran. E-mail: t_motamedifar@sadjad.ac.ir*
- *d.r Mahdi yaghoobi ,Department of Electrical Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran.. E-mail: yaghoobi@mshdiau.ac.ir*

Despite of complications of differential calculus, recent advancements in chaotic systems and the close relationship between fractals and fractional calculus have grown interests in applying it. Fractional calculus has a wider range than correct derivative. If we use fractional order instead of correct order derivative or integral, we should use fractional calculus for solving derivative and fractional integral.
Derivative-integrator operator is shown by ${}_aD_t^{\alpha}$. This operator is a sign for taking derivative and fractional

Grunwald–Letnikov, Riemann–Liouville, and Caputo are definitions that are applied for fractional derivatives.

$$ {}_aD_t^q = \begin{cases} \frac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \int_a^t (d\tau)^{-q} & q < 0 \end{cases} \quad (1) $$

Its Laplace transform is as follows:

$$ \int_0^\infty e^{-st} {}_0D_t^\alpha f(t)dt = s^\alpha F(s) \qquad \alpha \le 0 $$

$$ \int_0^\infty e^{-st} {}_0D_t^\alpha f(t)dt = s^\alpha F(s) - \sum_{k=0}^{n-1} s^k {}_0D_t^{\alpha-k-1} f(t)\Big|_{t=0} $$

$$ n-1 < \alpha \le n \in N \quad (2) $$

3. Problem Description

ISSN 2229-5518

Synchrony has a Greek root meaning to share the common time. The principle meaning of this word has been kept in ordinary application referring to settlement or affinity. Analysis of synchronization in dynamic systems has been a subject of considerable debate in physics as an important matter. The origin of this phenomenon relates back to 17th century when the second phase of pendulum clock hanging from a point and there was a weak coupling between them coordinate with each other. Later on, other samples of such phenomena were also observed. Recently, studies have extended to chaotic systems. As we know, chaotic systems are sensitive to primary conditions. Because of this property, these systems oppose to synchronization by nature. Even two totally similar systems starting to work with trivial differences gradually lose their coordination over times.

Chen's system is defined by the following equations:

$$\begin{aligned} {}_0D_t^{\alpha} x &= a(y-x) \\ {}_0D_t^{\alpha} y &= (c-a)x - xz + cy. \\ {}_0D_t^{\alpha} z &= xy - bz \end{aligned} \qquad (3)$$

Lou system is defined as follows:

$$\begin{cases} {}_0D_t^{\alpha} x = \rho(y-x) \\ {}_0D_t^{\alpha} y = -xz + \nu y. \\ {}_0D_t^{\alpha} z = xy - \mu z \end{cases} \qquad (4)$$

Here, the synchronization of two chaotic systems Chen-Lu is studied based on Lyapunov theorem and active sliding mode controller.

4. Designing Active Sliding Mode Controller

In this part, we obtained controller for follower-based fractional system using active sliding mode controller. Systems' order is the same. The method of designing the active sliding mode controller is a combination of active controller and sliding mode controller.

Consider a chaotic system with fractional order q ($0 < q < 1$). It is described by a fractional differential equation as follows:

$${}_0D_t^q x_1 = A_1x_1 + g_1(x_1) \qquad (5)$$

$\boldsymbol{x_1(t)} \in \boldsymbol{R^3}$ shows that the vector system is three-dimensional. $\boldsymbol{A_1} \in \boldsymbol{R^{3*3}}$ is the linear part of dynamic system and $\boldsymbol{g_1}: \boldsymbol{R^3} \rightarrow \boldsymbol{R^3}$ is the nonlinear part of the system. Relation 10 is the based system (Master). Controller $\boldsymbol{u(t)} \in \boldsymbol{R^3}$ is added to the follower system (slave) and obtained as follows:

$${}_0D_t^q x_1 = A_2x_2 + g_2(x_2) \qquad (6)$$

$\boldsymbol{x_2} \in \boldsymbol{R^3}$ is the three dimensional state vector of the follower system. And, $\boldsymbol{g_2}: \boldsymbol{R^3}, \boldsymbol{A_2} \in \boldsymbol{R^{3*3}}$ is the similar role of A1, g1 in the based system.

Synchronization means finding the control signal of $\boldsymbol{u(t)} \in \boldsymbol{R^3}$ to close the state of the follower system to the based state. To reach this goal, we can define the dynamic error of coordination as follows:

$${}_0D_t^q e = A_2x_2 + g_2(x_2) - A_1x_1 - g_1(x_1) + u(t) = A_e + G(x_1, x_2) + u(t) \qquad (7)$$

We have $e = x_1 - x_2$ and

$$G(x_1, x_2) = g_2(x_2) - g_1(x_1) + (A_2 - A_1)x_1 \qquad (8)$$

To simplify, we substitute that the linear part of the follower system with Matrix $A = A_2$. The main aim is to design controller $\boldsymbol{u(t)} \in \boldsymbol{R^3}$ to have:

$$lim_{t\rightarrow\infty} \|e(t)\| = 0 \qquad (9)$$

According to the method of designing active controller [6], the nonlinear part of dynamic error is eliminated by choosing the input vector:

$$u(t) = H(t) - G(x_1, x_2) \qquad (10)$$

The error of system (7) is rewritten as bellow:

$${}_0D_t^q e = A_e + H(t) \qquad (11)$$

The equation 11 describes the error dynamic by a new definition for the input of controller H (t). In active sliding mode controller, H(t) is developed based on sliding mode rules:

$$H(t) = kw(t) \qquad (12)$$

$K = [k_1, k_2, k_3]^T$ is the fixed gain vector and $\boldsymbol{w(t)} \in \boldsymbol{R^3}$ is the controller input which is defined as bellow:

ISSN 2229-5518

$$w(t) = \begin{cases} w^{+}(t)s(e) \geq 0 \\ w^{-}(t)s(e) \leq 0 \end{cases} \quad (13)$$

S = s € is the switching surface where dynamics are placed in its favorable zone. As a result, dynamic error is:

$${}_0D_t^q e = A_e + kw(t) \quad (14)$$

According to what was mentioned before, the good sliding controller is achieved based on developing the theory of sliding mode control [10-5].

1-4 Developing the Sliding Surface

The sliding surface can be determined as bellows:

$$s = Ce \quad (15)$$

C = [c1, c2, c3] is the fixed vector. By solving $\dot{S}(e) = 0$, which is the required condition, the equivalent control is achieved. S € = 0 is a condition for the state curve to remain in the switching level. Thus, in sliding mode controller, the two following conditions should be met:

$$\dot{S}(e) = 0 \quad , \quad s(e) = 0 \quad (16)$$

2-4 Developing Sliding Mode Controller

In designing the constant, we consider the relative convergent speed. Accordingly, the condition of reaching is chosen as follows:

$${}_0D_t^{1-q}s = -p\,sgn(s) - rs \quad (17)$$

Sgn (0) is the sign function. Gains p> 0 and r > 0 are determined to meet the sliding conditions and the sliding mode movement then occurs.

According to equations 14 and 17,

$${}_0D_t^{1-q}s = C_0D_t^q e = C[A_e + KW(t)] \quad (18)$$

Now from equations 17 and 18, the input control is determined:

$$(19) \quad w(t) = -(CK)^{-1}[C(rI + A)e + psgn(s)]$$

To define the optimum value r and p, we can use a generic algorithm. To do this, we employed particle swarm optimization (PSO).

5. PSO

The fit function for PSO is defined as follows[12]:

1. It is considered as desirable in terms of speed pursuit.
2. Based on an error index

$$Fitness = \int e1^2 + e2^2 + e3^2 dt \quad (20)$$

Table 1: parameters of particle swarm optimization

Number of Updates	50
Number of Primary Population	50
r	[0 100]
p	[0 100]
W	0.9
C_1	0.12
C_2	1.2

6. Numerical Simulation

Here, the numerical simulation results obtained by MATLAB for chaotic systems with similar order Chen-Lu:

1-6 Synchronization of two fractional order systems Lu and Chen

In this section, we apply active sliding mode controller by two fractional order systems Lu and Chen. Assume, Chen system follows Lu system. Accordingly Master and Slave systems are as follows:

$$\begin{cases} {}_0D_t^{0.9}x_1 = 35(y_1 - x_1) \\ {}_0D_t^{0.9}y_1 = -x_1z_1 + 28y_1, \quad (x_{10}, y_{10}, z_{10}) = (7, -4, 4) \\ {}_0D_t^{0.9}z_1 = x_1y_1 - 3z_1 \end{cases}$$

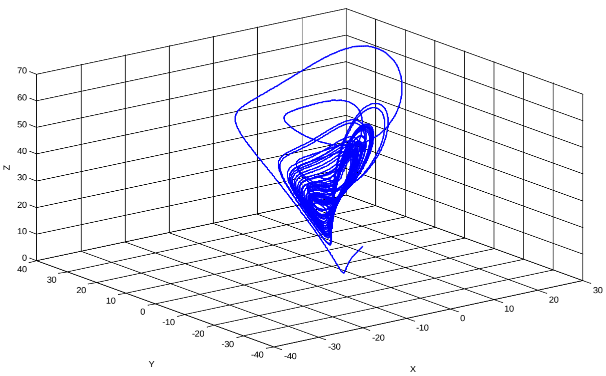
$$\begin{cases} {}_0D_t^{0.9}x_2 = 35(y_2 - x_2) \\ {}_0D_t^{0.9}y_2 = -7x_2 - x_2z_2 + 28y_2, \quad (x_{20}, y_{20}, z_{20}) = (1, 3, -1) \\ {}_0D_t^{0.9}z_2 = x_2y_2 - 3z_2 \end{cases}$$

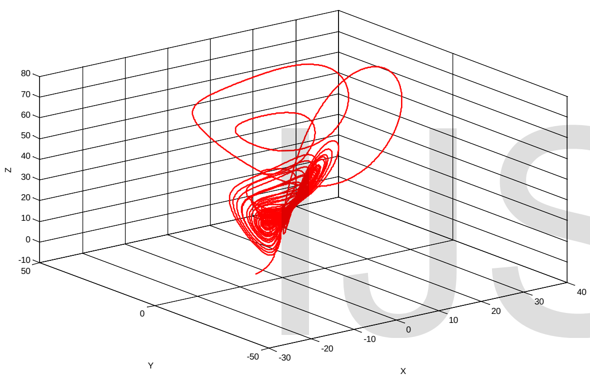
X_1, y_1, and z_1 are the variables of Master system and x_2, y_2 and z_2 are the variables of Slave system and q is the order of fractional derivative.

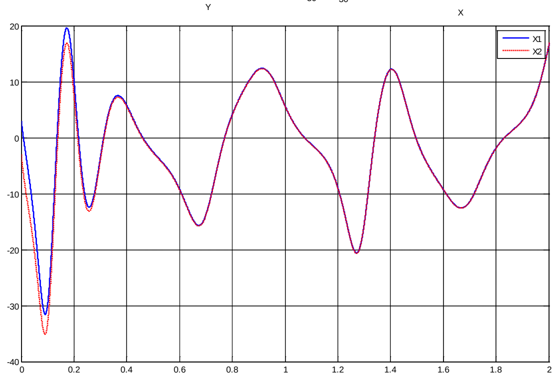
ISSN 2229-5518

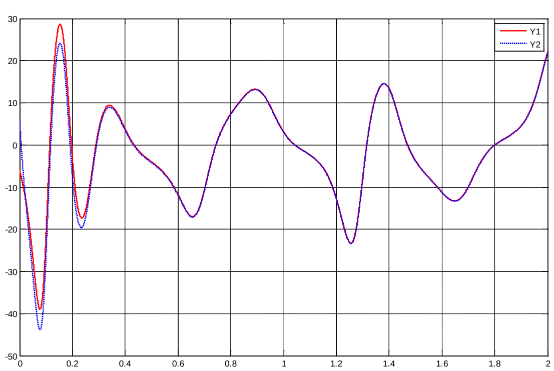
The purpose is to design the control signal in the condition of synchronization of two systems using active sliding mode controller.

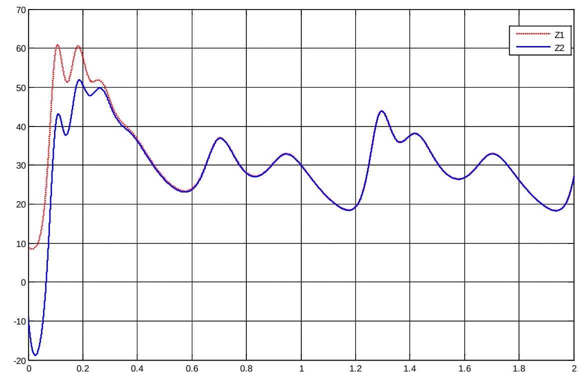
The simulation results for this state are as follows:

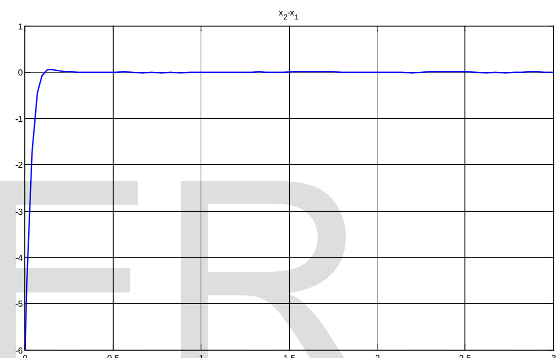


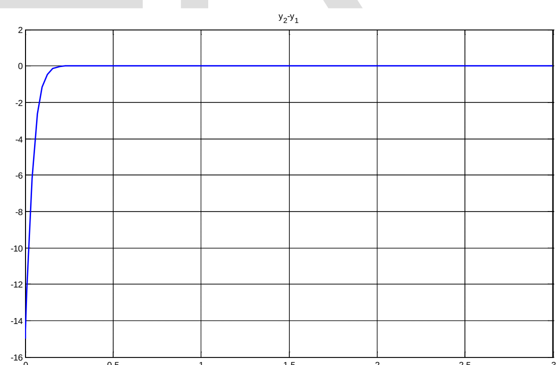


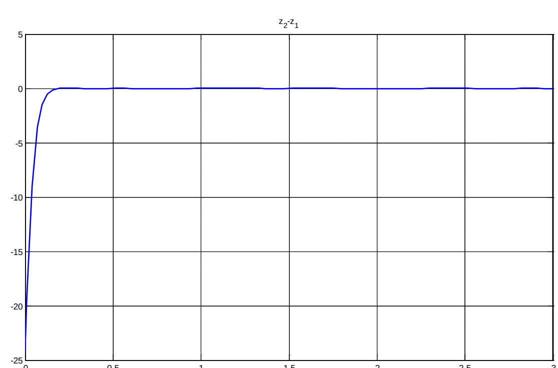












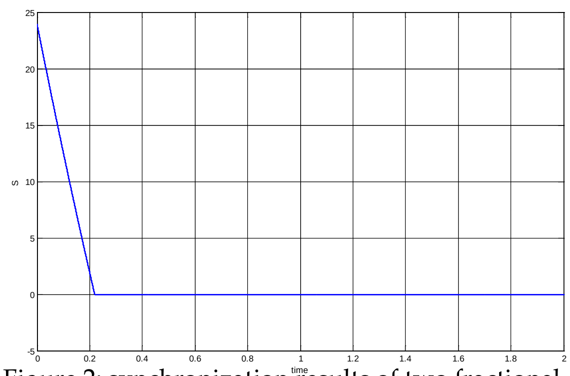


Figure 2: synchronization results of two fractional chaotic systems

5. Conclusion

Regarding the simulation results in Lu-Chen for q = 0.9, the usefulness of active sliding mode controller in fractional state is concluded. By an optimal chose, the controller parameters of based and follower systems have been synchronized. It is, thus, observed that all three states have been synchronized in less than 0.4 seconds which has been reduced comparing with similar methods.

References:

[1] *Negrete D.C., 2005, Fractional calculus: basic theory and applications (Part I), in: Lectures Presented at the Institute of Mathematics UNAM, Mexico.*

[2] *Wu, X.J., Li, J., Chen, G.R.: Chaos in the fractional order unified system and its synchronization. J. Franklin Inst.345, 392–401 (2008)*

[3] *Ge, Z.M., Ou, C.Y.: Chaos synchronization of fractional order modified Duffing systems with parameters excited by achaotic signal. Chaos Solitons Fractals35, 705–717 (2008)*

[4] *Tavazoei, M.S., Haeri, M.: Synchronization of chaotic fractional-order systems via active sliding mode controller.Phys.A,Stat.Mech.Appl.387, 57–70 (2008)*

[5] *Matouk, A.E.: Chaos, feedback control and synchronization of a fractional-order modified autonomous vander Pol–Duffing circuit. Commun. Nonlinear Sci. Numer.Simul.16, 975–986 (2011)*

[6] *Delavari, H., Ghaderi, R., Ranjbar, A., Momani, S.: Fuzzyfractional order sliding mode controller for nonlinear systems. Commun.Nonlinear Sci. Numer. Simul.15, 963–978(2010)*

[7] *Wu, X.J., Lu, H.T., Shen, S.L.: Synchronization of a newfractional-order hyperchaotic*

[8] *Wang, X.Y., He, Y.J., Wang, M.J.: Chaos control of a fractional order modified coupled dynamos system. NonlinearAnal.71, 6126–6134 (2009)*

[9] *Zheng, Y.G., Nian, Y.B., Wang, D.J.: Controlling fractional order chaotic systems based on Takagi–Sugeno fuzzy model and adaptive adjustment mechanism. Phys. Lett. A 375, 125–129 (2010)*

[10] *Dastranj, Mohammad Reza, Mojtdaba Rouhani, and Ahmad Hajipoor. "Design of Optimal Fractional Order PID Controller Using PSO Algorithm." International Journal of Computer Theory and Engineering 4.3 (2012): 429-432.*

[11] *Mohamad Reza Dastranj, Mohamad Reza Dastranj, Aminreza Riahi Aminreza Riahi, and Mahboubeh Moghaddas Mahboubeh Moghaddas. "Synchronization of Chaotic Fractional-Order Lu-Lu Systems with Active Sliding Mode Control." International Journal of Control and Automation 7.9 (2014): 131-138.*

[12] *Dastranj, M. R., M. Moghaddas, S. S. Afghu, and M. Rouhani. "PID control of inverted pendulum using particle swarm optimization (PSO) algorithm." In Communication Software and Networks (ICCSN), 2011 IEEE 3rd International Conference on, pp. 575-578. IEEE, 2011.*